

ΕΠΑΝΑΛΗΠΤΙΚΟ ΔΙΑΓΩΝΙΣΜΑ ΠΡΟΣΟΜΟΙΩΣΗΣ

ΤΑΞΗ: Γ' ΓΕΝΙΚΟΥ ΛΥΚΕΙΟΥ

ΣΑΒΒΑΤΟ 22 ΑΠΡΙΛΙΟΥ 2017

ΕΞΕΤΑΖΟΜΕΝΟ ΜΑΘΗΜΑ: ΜΑΘΗΜΑΤΙΚΑ ΚΑΙ ΣΤΟΙΧΕΙΑ ΣΤΑΤΙΣΤΙΚΗΣ ΓΕΝΙΚΗΣ ΠΑΙΔΕΙΑΣ

ΑΠΑΝΤΗΣΕΙΣ:

Θέμα 1^ο

A4

- i. Σ
- ii. Λ
- iii. Λ
- iv. Σ
- v. Σ

Θέμα 2^ο

i. $f(1) = a$

$$\begin{aligned} \lim_{x \rightarrow 1} f(x) &= \lim_{x \rightarrow 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x \rightarrow 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)} = \\ &= \lim_{x \rightarrow 1} \frac{x+3-2}{(x-1)(\sqrt{x+3}+2)} = \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)} = \\ &= \frac{1}{4} \end{aligned}$$

Επειδή f συνεχής $f(1) = \lim_{x \rightarrow 1} f(x) \Rightarrow a = \frac{1}{4}$

ii. $f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h)-f(1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{\sqrt{1+h+3}-2}{1+h-1} - \frac{1}{4}}{h} = \lim_{h \rightarrow 0} \frac{4\sqrt{h+4}-8-h}{h} =$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{4\sqrt{h+4}-(8+h)}{4h^2} = \lim_{h \rightarrow 0} \frac{(4\sqrt{h+4}-(8+h))(4\sqrt{h+4}+(8+h))}{4h^2(4\sqrt{h+4}+(8+h))} = \\ &= \lim_{h \rightarrow 0} \frac{(4\sqrt{h+4})^2-(8+h)^2}{4h^2(4\sqrt{h+4}+(8+h))} = \lim_{h \rightarrow 0} \frac{16(h+4)-(64+16h+h^2)}{4h^2(4\sqrt{h+4}+(8+h))} = \\ &= \lim_{h \rightarrow 0} \frac{16h+64-64-16h-h^2}{4h^2(4\sqrt{h+4}+(8+h))} = \lim_{h \rightarrow 0} \frac{-h^2}{4h^2(4\sqrt{h+4}+(8+h))} = \\ &= \frac{-1}{4(4\sqrt{4}+8)} = \frac{-1}{4(8+8)} = -\frac{1}{64} \end{aligned}$$

iii. $y = f'(1)x + \beta \Rightarrow y = -\frac{1}{64}x + \beta$

$$A\left(1, \frac{1}{4}\right) \in (\varepsilon): \frac{1}{4} = -\frac{1}{64} \cdot 1 + \beta \Rightarrow \frac{16}{4} + \frac{1}{64} = \beta \Rightarrow \beta = \frac{16}{64} + \frac{1}{64} = \frac{17}{64}$$

$$\text{Άρα, } (\varepsilon): y = -\frac{1}{64}x + \frac{17}{64}$$

Θέμα 3°

$$P(\Pi) = 80\% \quad P(M) = 40\% \quad P(\Pi \cap M) = 30\%$$

- i. $P(\Pi \cap M') = P(\Pi - M) = P(\Pi) - P(\Pi \cap M) = 80\% - 30\% = 50\%$
- ii. $P(\Pi \cup M) = P(\Pi) + P(M) - P(\Pi \cap M) = 80\% + 40\% - 30\% = 90\%$
- iii. $P(\Pi \cup M)' = 100\% - P(\Pi \cup M) = 100\% - 90\% = 10\%$
- iv. $P(\Pi \cap M)' = 100\% - P(\Pi \cap M) = 100\% - 30\% = 70\%$
- v. $P((\Pi - M) \cup (M - \Pi)) = P(\Pi - M) + P(M - \Pi) = P(\Pi) - P(\Pi \cap M) + P(M) - P(M \cap \Pi) = 80\% - 30\% + 40\% - 30\% = 60\%$

Θέμα 4°

A. $g(x) = x^3 - 6x^2 + 9x + 4$
 $g'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$
 $g'(x) = 0 \Rightarrow x^2 - 4x + 3 = 0$
 $\Delta = 16 - 12 = 4$

$$x_{1,2} = \frac{4 \pm 2}{2} = \begin{cases} \frac{4+2}{2} = 3 \\ \frac{4-2}{2} = 1 \end{cases}$$

x	$-\infty$	1	3	$+\infty$
g'	+	○	○	+
g	↗ ↘ ↗			

$$g \uparrow (-\infty, 1], [3, +\infty)$$

$$g \downarrow [1, 3]$$

Η g παρουσιάζει τοπικό μέγιστο στο $x_1 = 1$ με τιμή $g(1) = 1 - 6 + 9 + 4 = 8$

Η g παρουσιάζει τοπικό ελάχιστο στο $x_2 = 3$ με τιμή $g(3) = 3^3 - 6 \cdot 3^2 + 9 \cdot 3 + 4 = 27 - 54 + 27 + 4 = 4$

B.

x_i	v_i	$f_i\%$	N_i	$F_i\%$
1	3	6	$\beta=3$	6

2	$g(\alpha) = 8$	16	11	22
3	22	44	33	$22\beta = 66$
4	15	30	48	96
5	$\beta - \alpha = 2$	$g(\beta) = 4$	50	100
Σύνολο	50	100	-	-

$$f_5\% = \frac{v_5}{v} 100 \Rightarrow 4 = \frac{2}{v} 100 \Rightarrow \frac{4v}{4} = \frac{200}{4} \Rightarrow v = 50$$

$$v_1 = N_1 = 3$$

$$f_1\% = \frac{v_1}{v} 100 = \frac{3}{50} 100 = 6$$

$$F_1\% = f_1\% = 6$$

$$f_2\% = \frac{v_2}{v} 100 = \frac{8}{50} 100 = 16$$

$$N_2 = N_1 + v_2 = 3 + 8 = 11$$

$$F_2\% = F_1\% + f_2\% = 6\% + 16\% = 22\%$$

$$F_3\% = F_2\% + f_3\% \Leftrightarrow 66 = 22 + f_3\% \Rightarrow f_3\% = 44$$

$$f_3\% = \frac{v_3}{v} 100 \Leftrightarrow 44 = \frac{v_3}{50} 100 \Rightarrow v_3 = 22$$

$$N_3 = N_2 + v_3 = 11 + 22 = 33$$

$$v_1 + v_2 + v_3 + v_4 + v_5 = 50 \Leftrightarrow 33 + v_4 + 2 = 50 \Rightarrow v_4 = 15$$

$$f_4\% = \frac{v_4}{v} 100 = \frac{15}{50} 100 = 30$$

$$N_4 = N_3 + v_4 = 33 + 15 = 48$$

$$F_4\% = F_3\% + f_4\% = 66 + 30 = 96$$

$$i. \quad \bar{x} = \frac{\sum x_i v_i}{v} = \frac{1 \cdot 3 + 2 \cdot 8 + 3 \cdot 22 + 4 \cdot 15 + 2 \cdot 5}{50} = \frac{3 + 16 + 66 + 60 + 10}{50} = \frac{155}{50} = 3,1$$

$$ii. \quad \frac{v}{2} = \frac{50}{2} = 25$$

$$\delta = \frac{t_{25} + t_{26}}{2} = \frac{3+3}{2} = 3$$

$$iii. \quad y = a \cdot x + 9$$

$$\text{Αρα } \bar{y} = a \cdot \bar{x} + 9 \Leftrightarrow 40 = a \cdot 3,1 + 9 \Rightarrow \frac{3,1 \cdot a}{3,1} = \frac{31}{3,1} \Rightarrow a = 10$$

iv. Οι πρώτες 11 παρατηρήσεις είναι το 1 και το 2 κι αφού αυξάνονται κατά 3 μονάδες το 1 γίνεται 4 και το 2 γίνεται 5.

Ο καινούριος πίνακας συχνοτήτων είναι συχνοτήτων είναι

x_i	v_i	N_i
3	22	22
4	18	40
5	10	50
Σύνολο	50	-

$$\frac{v}{2} = \frac{50}{2} = 25$$

$$\delta = \frac{t_{25} + t_{26}}{2} = \frac{4+4}{2} = 4$$